

Hall current and suction/injection effects on the entropy generation of third grade fluid



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ABSTRACT

In this work, effects of Hall current and suction/injection on a steady, viscous, incompressible and electrically conducting third grade fluid past a semi-infinite plate with entropy generation is investigated. It is assumed that the fluid motion is induced by applied pressure gradient. Hot fluid is injected with a constant velocity at the injection wall while it is sucked off at the upper wall with the same velocity. The governing equations of Navier-Stokes, energy and entropy generation obtained are non-dimensionalised, the resulting dimensionless velocity and temperature profiles are solved by Adomian decomposition technique due to the nonlinearity of the coupled system of equations. The obtained solution for the velocity profile is validated by the exact solution and the existing one in literature at $M = 0$ and the analytical expressions for fluid velocity and temperature are utilized to calculate the entropy generation and irreversibility ratio. Various plots are presented and discussed. It is found that increasing Hall current parameter increases primary velocity, temperature, entropy generation and Bejan number while the reverse trend is observed when both suction/injection and magnetic field parameters are increased. It is also noticed that entropy production at the upper wall is due to heat transfer.

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1. Introduction

Recent advancement in technology has led to a renewed interest in the investigation of non-Newtonian fluids. These types of fluids cannot be described by the classical Navier-Stokes equation because they are of higher order and more complex, consequently various constitutive equations have been developed for the non-Newtonian fluids. The proposed models include Eyring-Powell model (Eldabe et al., 2003; Zueco and Bég, 2009; Rahimi et al., 2016); the couple stress model (Srinivasacharya and Srikanth, 2008; Srinivasacharya and Kaladhar, 2012; Adesanya and Makinde, 2012); second grade fluid model (Vajravelu and Roper, 1999; Siddiqui et al., 2003; Wenchang and Mingyu, 2004; Hayat et al., 2007) (which is the simplest subclass of non-Newtonian fluid, however this model has the limitation of not being able to predict the shear thinning/thickening properties) and the third grade fluid model

The study of magnetohydrodynamic flow has been extensively investigated in the past years as a result of its applications in plasma studies, MHD generators, nuclear reactor, metal purification, geothermal energy extractions, polymer technology and metallurgy. Numerous qualitative investigations with outstanding results have been conducted by various researchers such as Adesanya and Makinde (2012) used Eyring-Powell model to investigate heat transfer to magnetohydrodynamic non-Newtonian couple stress pulsatile flow between two parallel porous plates, Hassan and Gbadeyan (2015) examined a reactive hydromagnetic internal heat generating fluid flow through a channel. Shehzad et al. (2015) considered influence of convective heat and mass conditions in MHD flow of nanofluid. Mutuku-Njane and Makinde (2013) employed fourth-order Runge-Kutta with shooting technique to study the effects of buoyancy force and Navier slip on MHD flow of a Nanofluid over convectively heated vertical porous plates. Hayat et al. (2015) examined the effect of inclined magnetic field in flow of third grade fluid with variable thermal conductivity and submitted that higher values of magnetic field enhances the skin-friction, the temperature and concentration profiles of the fluid. Gbadeyan et al. (2010) considered the radiative

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effect on velocity, magnetic and temperature fields of a magnetohydrodynamic oscillatory flow past a limiting surface with variable suction. [Rashidi et al. \(2011\)](#) reported on hydromagnetic multi-physical flow phenomena from a rotating disk, differential transform method was applied, and it was shown that increase in magnetic parameter (M) suppresses radial velocity, decreases tangential velocity and elevates axial velocity. Furthermore, [Mohamed \(2009\)](#) studied the unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects. [Aydin and Kaya \(2008\)](#) investigated radiation effect on MHD mixed convection flow about a permeable vertical plate. Also, [Adesanya et al. \(2015a\)](#) examined hydromagnetic natural convection flow between vertical parallel plates with time-periodic boundary conditions.

All the above mentioned studies are limited only to applications where Hall Effect is negligible due to the assumption of small and moderate values of the magnetic field. The current trend of research is toward a strong magnetic field and a low density gas because of its numerous applications such as in space flight, nuclear fusion research, magnetohydrodynamic generators, refrigeration coils, electric transformers, Hall accelerators as well as biomedical engineering (such as cardiac MRI and ECG). Hall current occurs in a situation where the applied magnetic field is very strong or an ionized gas with low density leading to a reduction in conductivity normal to the magnetic field, as a result of the free spiraling of electrons and ions around the magnetic lines of force before collisions. This then induces a current in direction normal to both electric and magnetic fields. This is referred to as Hall Effect; the induced current is called Hall current. Several research work on this subject under various flow configurations are found in literature: [Raptis and Ram \(1984\)](#) studied the effects of hall current and rotation on the flow of electrically conducting rarefied gas, the work shows that as Hall parameter increases, the primary velocity increases near the plate and decreases away from the plate while the secondary velocity decreases. [Abd El-Aziz and Nabil \(2012\)](#) applied homotopy analysis method in the study of hydromagnetic mixed convection flow past an exponentially stretching sheet with Hall current. In [Das et al. \(2012\)](#), Hall effects on unsteady hydromagnetic flow induced by a porous plate was considered, it was observed that the primary velocity decreases whereas the secondary velocity increases with an increase in Hall parameter. Combined effects of Hall and ion-slip currents on unsteady MHD Couette flows in a rotating system was reported by [Jha and Apere \(2010\)](#), it was argued that Hall and ion slip parameters had a reducing effect on the magnitude of the secondary velocity, other results on Hall effect are found in [Pal et al. \(2012\)](#), [Asghar et al. \(2005\)](#), [Aboeldahab and Elbarbary \(2001\)](#), [Abo-Eldahab and El Aziz \(2004\)](#), [Ahmad et al. \(2010\)](#), [Ayub et al. \(2010\)](#).

The emphasis of this article is to investigate the influence of Hall current and suction/injection on the entropy generation of a third grade fluid. This subject is essential due to the fact that entropy generation occurs in moving fluid with high temperature; it is therefore pertinent to examine the effect of factors such as Hall current and suction/injection on entropy production. In this article the approach pioneered by [Bejan \(1982\)](#) and adopted by several other researchers such as [Adesanya et al. \(2015b\)](#), [Ajibade et al. \(2011\)](#), [Eegunjobi and Makinde \(2012\)](#), [Opanuga et al. \(2016, 2017a,b,c\)](#), [Bouabid et al. \(2011\)](#) is applied. Adomian decomposition technique is chosen for the analysis of the dimensionless governing equations because it is a powerful tool for handling highly nonlinear problems ([Opanuga et al., 2017d,e](#)). Moreover, the method is noted for its high accuracy and rapid convergence, it does not require any linearization, discretization, use of initial guess or perturbation.

The rest of this work is organized as follows: section 2 presents the flow analysis and non-dimensionalisation of the governing equations, in section 3 analytical solution by Adomian method is constructed, and in section 4 graphical results are presented and discussed based on the physics of the problem while section 5 concludes the work.

2. Mathematical formulation

Consider a steady, viscous, incompressible and electrically conducting fluid past a semi-infinite plate in the presence of a transversely imposed magnetic field with distance $2h$ apart. Let the coordinate system be such that the x – axis is taken along the lower plate in the flow direction, the y – axis is normal to the xy – plane while z – axis is perpendicular to the plates. A constant pressure gradient is induced in the flow direction; hot fluid is injected into the channel wall at the lower plate and sucked off at the upper plate with the same velocity. Following [Cowling \(1957\)](#) the generalized Ohm's law taking the effect of Hall current into account is

$$J + \frac{\omega_e \tau_e}{B_0} (J \times B) = \sigma(E + q \times B) \quad (1)$$

Furthermore, it is assumed that if $J = j_x j_y j_z$, are the components of the current density J , the equations of conservation of electric charge $\nabla \cdot J = 0$ shows that j_z is constant which is assumed to be zero because $j_z = 0$ at the plates which are electrically non-conducting. It then implies that $j_z = 0$, everywhere in the flow. Furthermore, the electrical field $E = 0$ following [Meyer \(1958\)](#). Under these assumptions equation (1) yields

$$j_x + m j_y = -\sigma u B_0 \quad (2)$$

$$j_y - m j_x = -\sigma u B_0 \quad (3)$$

where $m = \omega_e \tau_e$ represents the Hall parameter.

Solving equations (2) and (3) for j_x and j_y yields

$$j_x = \frac{\sigma B_0}{1+m^2} (w + mu) \quad (4)$$

$$j_y = \frac{\sigma B_0}{1+m^2} (mw - u) \quad (5)$$

The governing equations for the flow under Boussinesq's approximation following Das and Jana (2013) and Adesanya et al. (2017) are:

Navier-Stoke equation along x – axis

$$\rho v_0 \frac{du^*}{dy^*} = -\frac{dp}{dx} + \mu \frac{d^2 u^*}{dy^{*2}} + 6\beta_3 \frac{d^2 u^{*2}}{dy^{*2}} \left(\frac{du^*}{dy^*} \right)^2 - \frac{\sigma B_0}{1+m^2} (u^* - mw^*); u^*(-h) = 0 = u^*(h). \quad (6)$$

Navier-Stoke equation along y – axis

$$\rho v_0 \frac{dw^*}{dy^*} = \mu \frac{d^2 w^*}{dy^{*2}} + 6\beta_3 \frac{d^2 w^{*2}}{dy^{*2}} \left(\frac{dw^*}{dy^*} \right)^2 - \frac{\sigma B_0}{1+m^2} (w^* + mu^*); w^*(-h) = 0 = w^*(h). \quad (7)$$

Energy equation

$$\rho c_p \frac{dT^*}{dy^*} = k \frac{d^2 T^*}{dy^{*2}} + \mu \left(\left(\frac{du^*}{dy^*} \right)^2 + \left(\frac{dw^*}{dy^*} \right)^2 \right) + 2\beta_3 \left(\left(\frac{du^*}{dy^*} \right)^2 + \left(\frac{dw^*}{dy^*} \right)^2 \right) - \lambda \sigma B_0^2 (w^{*2} + u^{*2}); T^*(-h) = 0 = T^*(h). \quad (8)$$

The following dimensionless variables are introduced

$$y = \frac{y^*}{h}, u = \frac{u^*}{U}, w = \frac{w^*}{U}, \gamma = \frac{\beta_3 2}{\mu h^2}, \theta = \frac{T^* - T_0}{T_h - T_0}, A = -\frac{h^2}{\mu U} \frac{dp}{dx}, \nu = -\frac{\mu}{\rho}, Br = -\frac{\mu U^2}{k(T_h - T_0)}, N_s = \frac{T_0^2 h^2 E_G}{k(T_h - T_0)^2}, \Omega = \frac{k T_h - T_0}{T_0}, s = \frac{v_0 h}{\nu}, Pr = \frac{k \rho c_p}{k} \quad (9)$$

Using the above non-dimensional variables (9) in equations (6-8) yields

$$s \frac{du}{dy} = A + \frac{d^2 u}{dy^2} + 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 - \frac{M^2}{1+m^2} (u - mw); u(-1) = 0 = u(1) \quad (10)$$

$$s \frac{dw}{dy} = \frac{d^2 w}{dy^2} + 6\gamma \frac{d^2 w}{dy^2} \left(\frac{dw}{dy} \right)^2 - \frac{M^2}{1+m^2} (w + mu); w(-1) = 0 = w(1) \quad (11)$$

$$sPr \frac{d\theta}{dy} = \frac{d^2 \theta}{dy^2} + Br \left(\left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 \right) + 2\gamma \left[\left(\frac{dw}{dy} \right)^4 + \left(\frac{du}{dy} \right)^4 \right] + \lambda M^2 (w^2 + u^2); \theta(-1) = 0 = \theta(1) \quad (12)$$

Positive value of s (suction/injection parameter) indicates injection of hot fluid at the lower wall and suction at the upper wall.

3. Solution by Adomian decomposition method

To apply Adomian decomposition method, equations (10-12) are written in integral form as

$$u(y) = a_0 + \int_0^y \int_0^y \left\{ s \frac{du}{dy} - A - 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 + \frac{M^2}{1+m^2} (u - mw) \right\} dY dY \quad (13)$$

$$u(y) = a_1 + \int_0^y \int_0^y \left\{ s \frac{dw}{dy} - 6\gamma \frac{d^2 w}{dy^2} \left(\frac{dw}{dy} \right)^2 + \frac{M^2}{1+m^2} (w + mu) \right\} dY dY \quad (14)$$

and

$$\theta(y) = b + \int_0^y \int_0^y \left\{ sPr \frac{d\theta}{dy} - Br \left\{ \left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 \right\} + 2\gamma \left[\left(\frac{du}{dy} \right)^4 + \left(\frac{dw}{dy} \right)^4 \right] + \lambda M^2 (w^2 + u^2) \right\} dY dY. \quad (15)$$

By ADM, an infinite series solution is defined as

$$u(y) = \sum_{n=0}^{\infty} u_n(y), w(y) = \sum_{n=0}^{\infty} w_n(y), \theta(y) = \sum_{n=0}^{\infty} \theta_n(y). \quad (16)$$

Using (16) in equations (13-15) yields

$$\sum_{n=0}^{\infty} u_n(y) = a_0 + \int_0^y \int_0^y \left\{ s \frac{du}{dy} - A - 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 + \frac{M^2}{1+m^2} (u - mw) \right\} dY dY \quad (17)$$

$$\sum_{n=0}^{\infty} w_n(y) = a_1 + \int_0^y \int_0^y \left\{ s \frac{dw}{dy} - 6\gamma \frac{d^2 w}{dy^2} \left(\frac{dw}{dy} \right)^2 + \frac{M^2}{1+m^2} (w + mu) \right\} dY dY \quad (18)$$

and

$$\sum_{n=0}^{\infty} \theta_n(y) = b + \int_0^y \int_0^y \left\{ sPr \frac{d\theta}{dy} - Br \left\{ \left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 \right\} + 2\gamma \left[\left(\frac{du}{dy} \right)^4 + \left(\frac{dw}{dy} \right)^4 \right] + \lambda M^2 (w^2 + u^2) \right\} dY dY \quad (19)$$

The zeroth order term of (17-19) are of the form

$$\sum_{n=0}^{\infty} u_n(y) = a_0 + \int_0^y \int_0^y \{A\} dY dY, \sum_{n=0}^{\infty} u_n(y) = a_1 \quad (20)$$

$$\sum_{n=0}^{\infty} \theta_n(y) = b. \quad (21)$$

The following recurrence relations are used to obtain other terms

$$\sum_{n=0}^{\infty} u_{n+1}(y) = \int_0^y \int_0^y \left\{ s \frac{du}{dy} - 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 + \frac{M^2}{1+m^2} (u - mw) \right\} dY dY, \quad (22)$$

$$\sum_{n=0}^{\infty} w_{n+1}(y) = \int_0^y \int_0^y \left\{ s \frac{dw}{dy} - 6\gamma \frac{d^2 w}{dy^2} \left(\frac{dw}{dy} \right)^2 + \frac{M^2}{1+m^2} (w + mu) \right\} dY dY, \quad (23)$$

and

$$\sum_{n=0}^{\infty} \theta_{n+1}(y) = \int_0^y \int_0^y \left\{ sPr \frac{d\theta}{dy} - Br \left\{ \left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 \right\} + 2\gamma \left[\left(\frac{du}{dy} \right)^4 + \left(\frac{dw}{dy} \right)^4 \right] + \lambda M^2 (w^2 + u^2) \right\} dY dY. \quad (24)$$

The non-linear terms of equation (22-24) can be written as

$$A_n = \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2, B_n = \left(\frac{du}{dy} \right)^2, C_n = u^2 \quad (25)$$

while the Adomian polynomials are computed as

$$A_0 = \frac{d^2 u_0}{dy^2} \left(\frac{du_0}{dy} \right)^2, A_1 = 2 \frac{d^2 u_0}{dy^2} \left(\frac{du_0}{dy} \right)^2 \left(\frac{du_1}{dy} \right) + \frac{d^2 u_1}{dy^2} \left(\frac{du_0}{dy} \right)^2 \quad (26)$$

$$A_2 = \frac{d^2 u_0}{dy^2} \left(\frac{du_1}{dy} \right)^2 + 2 \frac{d^2 u_1}{dy^2} \left(\frac{du_0}{dy} \right) \left(\frac{du_2}{dy} \right) + 2 \frac{d^2 u_1}{dy^2} \quad (27)$$

$$B_0 = \left(\frac{du_0}{dy}\right)^2, B_1 = 2\left(\frac{du_0}{dy}\right)^2 \frac{du_1}{dy}, \frac{du_0}{dy}, B_2 = \left(\frac{du_1}{dy}\right)^2 + 2\frac{du_0}{dy} \frac{du_2}{dy} \quad (28)$$

$$C_0 = u_0^2, C_1 = u_0 u_1, C_2 = u_1^2 + 2u_0 u_2. \quad (29)$$

Using (26-29) in equations (22-24) gives the recursive relations as

$$\sum_{n=0}^{\infty} u_{n+1}(y) = \int_0^y \int_0^y \left\{ s \frac{du}{dy} - 6\gamma A1_n + \frac{M^2}{1+m^2} (u - mw) \right\} dY dY, \quad (30)$$

$$\sum_{n=0}^{\infty} w_{n+1}(y) = \int_0^y \int_0^y \left\{ s \frac{dw}{dy} - 6\gamma A2_n + \frac{M^2}{1+m^2} (w + mu) \right\} dY dY, \quad (31)$$

and

$$\sum_{n=0}^{\infty} \theta_{n+1}(y) = \int_0^y \int_0^y \left\{ s Pr \frac{d\theta}{dy} - Br B1_n (1 + 2\gamma B1_n) - Br B2_n (1 + 2\gamma B2_n) - Br \{ \lambda M^2 (C1_n + C2_n) \} \right\} dY dY. \quad (32)$$

Equations (30-32) are coded in Mathematica software to obtain the solution. The graphical results are shown in Figs. 1-12.

3.1. Entropy generation

The local volumetric entropy generation rate for the flow according to Bejan (1982) is shown below,

$$E_G = \frac{k}{T_0^2} \left(\frac{dT^*}{dy^*} \right)^2 + \frac{\mu}{T_0} \left[\left(\frac{du^*}{dy^*} \right)^2 + \left(\frac{dw^*}{dy^*} \right)^2 + \frac{2\beta_3}{\mu} \left(\left(\frac{du^*}{dy^*} \right)^4 + \left(\frac{dw^*}{dy^*} \right)^4 \right) + \lambda \frac{\sigma B_0^2}{\rho v} (w^{*2} + u^{*2}) \right]. \quad (33)$$

Equation (33) shows that heat transfer, fluid friction and magnetic field contribute to entropy generation. Substituting equation (9) in equation (33) yields the dimensionless entropy generation expression for the flow as

$$Ns = \left(\frac{d\theta}{dy} \right)^2 + \frac{Br}{\Omega} \left[\left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 + \left(2\gamma \left(\frac{du}{dy} \right)^4 + \left(\frac{dw}{dy} \right)^4 \right) + \lambda M^2 (w^2 + u^2) \right]. \quad (34)$$

Irreversibility ratio is represented as shown below

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\Phi}, \Phi = \frac{N_2}{N_1} \quad (35)$$

where

$$N_1 = \left(\frac{d\theta}{dy} \right)^2, N_2 = \frac{Br}{\Omega} \left[\left(\frac{du}{dy} \right)^2 + \left(\frac{dw}{dy} \right)^2 + \left(2\gamma \left(\frac{du}{dy} \right)^4 + \left(\frac{dw}{dy} \right)^4 \right) + \lambda M^2 (w^2 + u^2) \right] \quad (36)$$

Note that N_1 is the irreversibility resulting from heat transfer while N_2 is entropy generation due to viscous dissipation magnetic field. The determination of the source of irreversibility that dominates entropy generation is evident from equation (35). Heat irreversibility dominates when $Be = 1$ and $Be = 0$ corresponds to when viscous dissipation is the dominant contributor to entropy

generation; while $Be = 0.5$ is assigned when both contribute equally.

4. Results and discussion

In this article, Hall current effect on the entropy generation rate of a third grade fluid with suction/injection has been investigated. The results are presented as follows in Figs. 1-12.

4.1. Effect of thermophysical parameters on velocity profile

In Figs. 1a and 1b, the effects of Hall current on fluid velocity is displayed, it is noticed that primary velocity is enhanced with an increase in Hall parameter whereas secondary velocity reduces. Figs. 2a and 2b display the influence of suction/injection on fluid motion; the plots reveal that both primary and secondary velocities reduce as suction/injection parameter increases. This indicates that the net effect of suction is to decrease fluid motion as submitted by Pop and Watanabe (1992) and Uwanta and Hamza (2014).

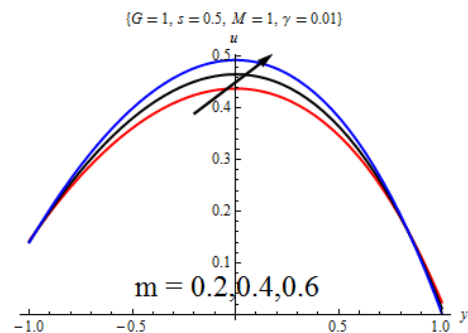


Fig. 1a: Primary velocity vs. hall current

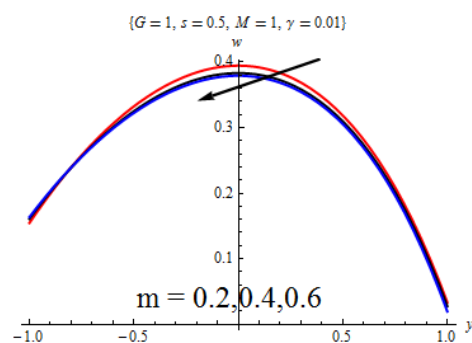


Fig. 1b: Secondary velocity vs. hall current

Furthermore, Figs. 3a and 3b display the effect of magnetic field parameter on fluid velocity; from the plots a rise in the magnetic field parameter suppresses fluid motion for both primary and secondary velocities. This is an indication of the dampening effect of the transverse magnetic field on fluid velocity which inhibits fluid motion.

4.2. Effect of thermophysical parameters on temperature profile

In this section, the influence of thermophysical parameters such as Hall current, suction/injection

and magnetic field parameters on fluid temperature are discussed.

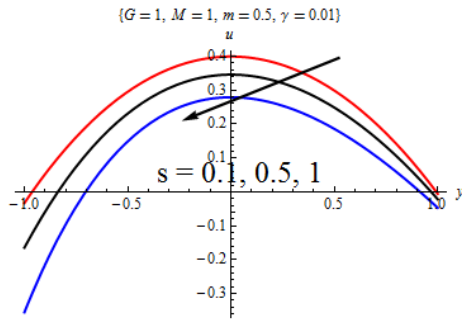


Fig. 2a: Primary velocity vs. suction/injection

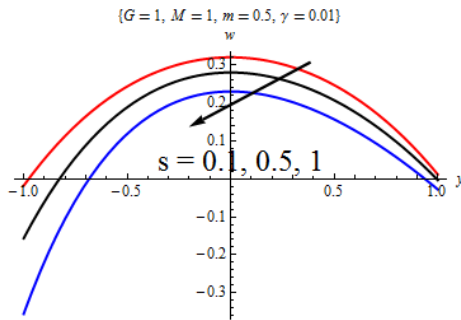


Fig. 2b: Secondary velocity vs. suction/injection

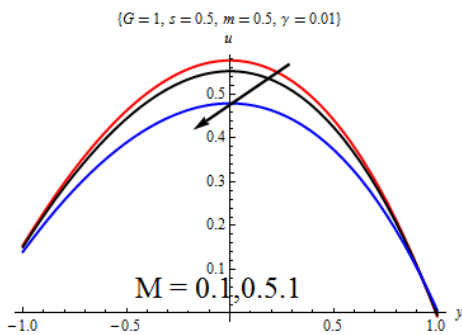


Fig. 3a: Primary velocity vs. Hartman number

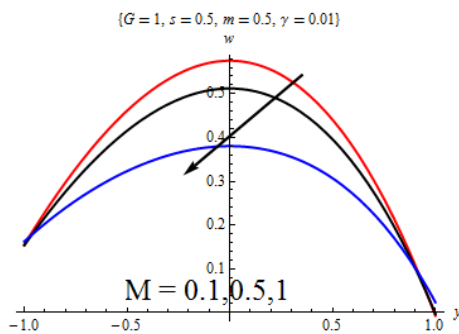


Fig. 3b: Secondary velocity vs. Hartman number

Fig. 4 shows that fluid temperature is enhanced as Hall parameter value increases; the effect is more significant in the middle of the channel than at the channel walls as observed from the plot. In Fig. 5, increase in suction/injection parameter raises fluid temperature at the lower and middle channels while the effect is less significant between plate $y=0.6$ and $y=1$. Injection of hot fluid into the channel at the lower wall and blowing at the upper wall is attributed to this observation. Further, it is

observed in Fig. 6 that fluid temperature increases with increasing values of Hartman number, the rise is due to the Lorentz heating which is present in the flow.

4.3. Effect of thermophysical parameters on entropy generation

Effects of Hall current, suction/injection and magnetic field parameters on entropy generation rate are depicted in Figs. 7-9.

Fig. 7 indicates that entropy generation increases with increase in Hall current both at the walls and center of the channel. Fig. 8 demonstrates the influence of suction/injection parameter on entropy generation, it is indicated that entropy is enhanced at the lower wall with injection of hot fluid whereas a reverse phenomenon is observed at the upper wall. Fig. 9 presents the effect of magnetic field parameter on the entropy generation rate. It is interesting to note that entropy production is suppressed at the channel walls but increases in the middle of the channel.

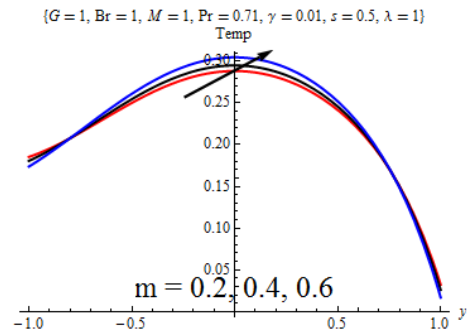


Fig. 4: Temperature vs. hall current

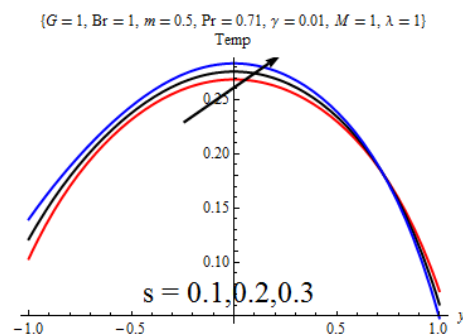


Fig. 5: Temperature vs. suction/injection

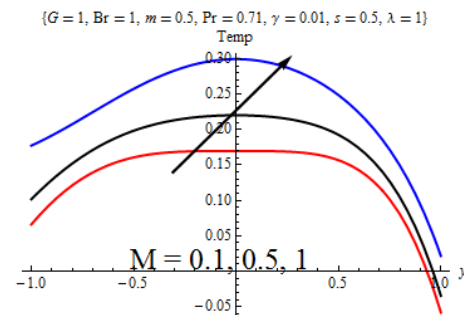


Fig. 6: Temperature vs. Hartman number

4.4. Effect of thermophysical parameters on Bejan number

In Figs. 10-12 the response of Bejan number to variation of Hall, suction/injection and magnetic field parameters are presented. Generally it is observed that Bejan number increases at the upper wall while it is either not significant or reduced at the lower wall. It is noted from the above that entropy generation at the upper plate is due to heat transfer.

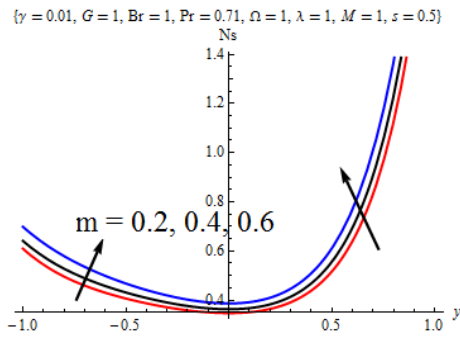


Fig. 7: Entropy generation vs. hall current

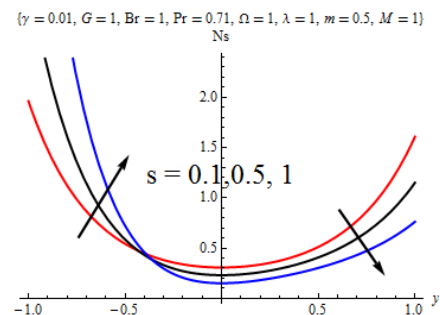


Fig. 8: Entropy generation vs. suction/injection

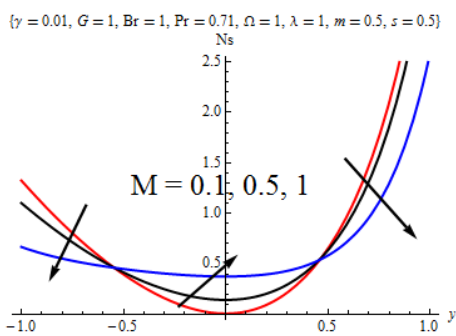


Fig. 9: Entropy generation vs. Hartman number

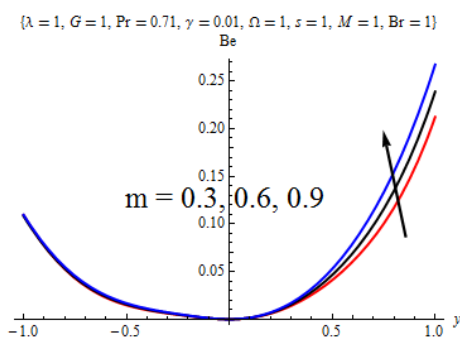


Fig. 10: Bejan number vs. hall current

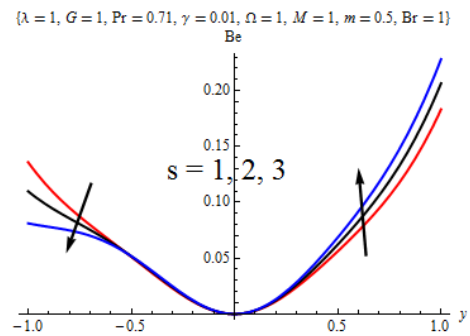


Fig. 11: Bejan number vs. suction/injection

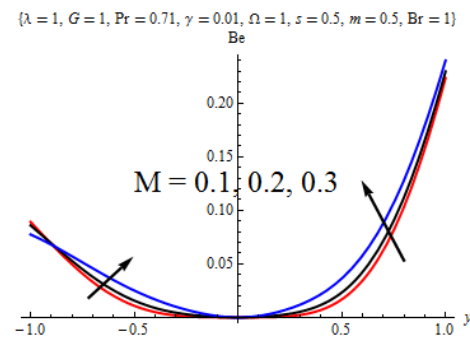


Fig. 12: Bejan number vs. Hartman number

5. Conclusion

In this article, Hall current and suction/injection effects on the entropy generation rate of third grade fluid is investigated. Graphs are presented to explain the obtained results and base on the results, the following conclusions are made:

1. Increase in Hall current parameter increases primary velocity, fluid temperature, entropy generation and Bejan number,
2. Suction/injection reduces both primary and secondary velocities, increases fluid temperature while Bejan number is enhanced only at the upper plate,
3. Magnetic field parameter inhibits fluid velocity, raises fluid temperature, entropy generation (in the middle of the channel) and Bejan number (at channel walls),
4. Generally entropy generation at the upper wall is due to heat transfer.

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Nomenclature

A	dimensionless pressure gradient,
Be	Bejan number,
Br	Brinkman number,
E_G	local volumetric entropy generation rate,
k	thermal conductivity of the fluid,
Ns	dimensionless entropy generation parameter,

p	fluid pressure,
s	suction/injection parameter ($s > 0$) is suction and $s < 0$ is injection),
T'	fluid temperature,
T_0	initial fluid temperature,
u, w	the dimensionless velocities,
u'	velocity component along x-axis,
U	characteristic velocity,
ν	kinematic viscosity,
v_0	constant velocity of fluid suction/injection,
w'	velocity component along y-axis

Greek letters

β_3	material coefficient,
γ	dimensionless third grade material parameter,
μ	dynamic viscosity,
θ	dimensionless temperature,
Ω	temperature difference parameter,
ψ	heat transfer coefficient.

References

- Abd El-Aziz M and Nabil T (2012). Homotopy analysis solution of hydromagnetic mixed convection flow past an exponentially stretching sheet with Hall current. *Mathematical Problems in Engineering*, 2012: Article ID 454023, 26 Pages. <https://doi.org/10.1155/2012/454023>
- Abo-Eldahab EM and El Aziz MA (2004). Hall current and ohmic heating effects on mixed convection boundary layer flow of a micropolar fluid from a rotating cone with power-law variation in surface temperature. *International Communications in Heat and Mass Transfer*, 31(5): 751-762.
- Aboeldahab EM and Elbarbary EM (2001). Hall current effect on magnetohydrodynamic free-convection flow past a semi-infinite vertical plate with mass transfer. *International Journal of Engineering Science*, 39(14): 1641-1652.
- Adesanya SO and Makinde OD (2012). Heat transfer to magnetohydrodynamic non-Newtonian couple stress pulsatile flow between two parallel porous plates. *Zeitschrift für Naturforschung A*, 67(10-11): 647-656.
- Adesanya SO, Falade JA, Jangili S, and Bég OA (2017). Irreversibility analysis for reactive third-grade fluid flow and heat transfer with convective wall cooling. *Alexandria Engineering Journal*, 56(1): 153-160.
- Adesanya SO, Kareem SO, Falade JA, and Arekete SA (2015a). Entropy generation analysis for a reactive couple stress fluid flow through a channel saturated with porous material. *Energy*, 93: 1239-1245.
- Adesanya SO, Oluwadare EO, Falade JA, and Makinde OD (2015b). Hydromagnetic natural convection flow between vertical parallel plates with time-periodic boundary conditions. *Journal of Magnetism and Magnetic Materials*, 396: 295-303.
- Ahmad M, Zaman H, and Rehman N (2010). Effects of hall current on unsteady MHD flows of a second grade fluid. *Central European Journal of Physics*, 8(3): 422-431.
- Ajibade AO, Jha BK, and Omame A (2011). Entropy generation under the effect of suction/injection. *Applied Mathematical Modelling*, 35(9): 4630-4646.
- Asghar S, Mohyuddin MR, and Hayat T (2005). Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating disks. *International journal of Heat and Mass transfer*, 48(3-4): 599-607.
- Aydin O and Kaya A (2008). Radiation effect on MHD mixed convection flow about a permeable vertical plate. *Heat and Mass Transfer*, 45(2): 239-246.
- Ayub M, Zaman H, and Ahmad M (2010). Series solution of hydromagnetic flow and heat transfer with Hall effect in a second grade fluid over a stretching sheet. *Open Physics*, 8(1): 135-149.
- Bejan A (1982). *Entropy generation through heat and fluid flow*. Wiley, New York, USA.
- Bouabid M, Magherbi M, Hidouri N, and Brahim AB (2011). Entropy generation at natural convection in an inclined rectangular cavity. *Entropy*, 13(5): 1020-1033.
- Cowling TG (1957). *Magnetohydrodynamics*. Interscience Tracts Physics and Astronomy, 4: 24-27.
- Das S and Jana RN (2013). Effect of hall current on entropy generation in porous channel with suction/injection. *International Journal of Energy and Technology*, 5(25): 1-11.
- Das S, Maji SL, and Jana RN (2012). Hall effects on unsteady hydromagnetic flow induced by a porous plate. *International Journal of Computer Applications*, 57(18): 37-44.
- Egunjobi AS and Makinde OD (2012). Effects of Navier slip on entropy generation in a porous channel with suction/injection. *Journal of Thermal Science and Technology*, 7(4): 522-535.
- Eldabe NTM, Hassan AA, and Mohamed MA (2003). Effect of couple stresses on the MHD of a non-Newtonian unsteady flow between two parallel porous plates. *Zeitschrift für Naturforschung A*, 58(4): 204-210.
- Gbadeyan JA, Idowu AS, Areo AO, and Olaleye. OP (2010). The radiative effect on velocity, magnetic and temperature fields of a magneto hydrodynamic oscillatory flow past a limiting surface with variable suction. *Journal of Mathematical Sciences*, 21: 395-411.
- Hassan AR and Gbadeyan JA (2015). A reactive hydromagnetic internal heat generating fluid flow through a channel. *International Journal of Heat and Technology*, 33(3): 43-50.
- Hayat T, Abbas Z, Sajid M, and Asghar S (2007). The influence of thermal radiation on MHD flow of a second grade fluid. *International Journal of Heat and Mass Transfer*, 50(5-6): 931-941.
- Hayat T, Shafiq A, Alsaedi A, and Asghar S (2015). Effect of inclined magnetic field in flow of third grade fluid with variable thermal conductivity. *AIP Advances*, 5(8): 087108.
- Jha BK and Apere CA (2010). Combined effect of hall and ion-slip currents on unsteady mhd couette flows in a rotating system. *Journal of the Physical Society of Japan*, 79(10): 1-9.
- Meyer RC (1958). On reducing aerodynamic heat-transfer rates by magnetohydrodynamic techniques. *Journal of the Aerospace Sciences*, 25(9): 561-566.
- Mohamed RA (2009). Double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects. *Applied Mathematical Sciences*, 3(13): 629-651.
- Mutuku-Njane WN and Makinde OD (2013). Combined effect of Buoyancy force and Navier slip on MHD flow of a nanofluid over a convectively heated vertical porous plate. *The Scientific World Journal*, 2013: Article ID 725643, 8 Pages. <https://doi.org/10.1155/2013/725643>
- Opanuga AA, Gbadeyan JA, and Iyase SA (2017a). Second law analysis of hydromagnetic couple stress fluid embedded in a non-Darcian porous medium. *International Journal of Applied Mathematics*, 47(3): 287-294.
- Opanuga AA, Gbadeyan JA, Iyase SA, and Okagbue HI (2016). Effect of thermal radiation on the entropy generation of hydromagnetic flow through porous channel. *The Pacific Journal of Science and Technology*, 17(2): 59-68.

- Opanuga AA, Okagbue HI, Agboola OO, and Imaga OF (2017b). Entropy generation analysis of buoyancy effect on hydromagnetic poiseuille flow with internal heat generation. *Defect and Diffusion Forum*, 378: 102-112.
- Opanuga AA, Okagbue HI, and Agboola OO (2017c). Irreversibility analysis of a radiative MHD Poiseuille Flow through Porous Medium with slip condition. In the *World Congress on Engineering 2017*, London, UK, 1: 1-5.
- Opanuga AA, Owoloko EA, Agboola OO, and Okagbue HI (2017d). Application of homotopy perturbation and modified Adomian decomposition methods for higher order boundary value problems. In the *World Congress on Engineering 2017*, London, UK, 1: 1-5.
- Opanuga AA, Owoloko EA, and Okagbue HI (2017e). Comparison homotopy perturbation and adomian decomposition techniques for parabolic equations. In *The World Congress on Engineering and Computer Science*, San Francisco, USA: 876-882.
- Pal D, Talukdar B, Shivakumara IS, and Vajravelu K (2012). Effects of hall current and chemical reaction on oscillatory mixed convection-radiation of a micropolar fluid in a rotating system. *Chemical Engineering Communications*, 199(8): 943-965.
- Pop I and Watanabe T (1992). The effects of suction or injection in boundary layer flow and heat transfer on a continuous moving surface. *Technische Mechanik*, 13: 49-54.
- Rahimi J, Ganji DD, Khaki M, and Hosseinzadeh K (2016). Solution of the boundary layer flow of an Eyring-Powell non-Newtonian fluid over a linear stretching sheet by collocation method. *Alexandria Engineering Journal*, 56(4): 621-627.
- Raptis A and Ram PC (1984). Effects of hall current and rotation. *Astrophysics and space science*, 106(2): 257-264.
- Rashidi MM, Erfani E, Bég OA, and Ghosh SK (2011). Modified differential transform method (DTM) simulation of hydromagnetic multi-physical flow phenomena from a rotating disk. *World Journal of Mechanics*, 1(05): 217-230.
- Shehzad SA, Hayat T, and Alsaedi A (2015). Influence of convective heat and mass conditions in MHD flow of nanofluid. *Bulletin of the Polish Academy of Sciences Technical Sciences*, 63(2): 465-474.
- Siddiqui AM, Mohyuddin MR, Hayat T, and Asghar S (2003). Some more inverse solutions for steady flows of a second-grade fluid. *Archives of Mechanics*, 55(4): 373-387.
- Srinivasacharya D and Kaladhar K (2012). Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects. *Communications in Nonlinear Science and Numerical Simulation*, 17(6): 2447-2462.
- Srinivasacharya D and Srikanth D (2008). Effect of couple stresses on the pulsatile flow through a constricted annulus. *Comptes Rendus Mecanique*, 336(11-12): 820-827.
- Uwanta IJ and Hamza MM (2014). Effect of suction/injection on unsteady hydromagnetic convective flow of reactive viscous fluid between vertical porous plates with thermal diffusion. *International Scholarly Research Notices*, 2014: Article ID 980270, 14 Pages. <https://doi.org/10.1155/2014/980270>
- Vajravelu K and Roper T (1999). Flow and heat transfer in a second grade fluid over a stretching sheet. *International Journal of Non-Linear Mechanics*, 34(6): 1031-1036.
- Wen Chang T and Mingyu X (2004). Unsteady flows of a generalized second grade fluid with the fractional derivative model between two parallel plates. *Acta Mechanica Sinica*, 20(5): 471-476.
- Zueco J and Bég OA (2009). Network numerical simulation applied to pulsatile non-Newtonian flow through a channel with couple stress and wall mass flux effects. *International Journal of Applied Mathematics and Mechanics*, 5(2): 1-16.